Currency Premia in Open Economies

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Abstract

We develop a two-country international asset pricing model in which investors are heterogeneous. Goods and asset markets are perfectly integrated, nonetheless a currency risk premium arises: uncovered interest parity is violated, and exchange rate dynamics are consistent with the empirical finding of negative skewness. Countries' interest rates are driven by the expected growth in supply and demand of local goods. High growth in a country's output relative to demand raises the interest rate. The currency premium reflects the associated risk of international trade. If foreign demand for a country's good is driven by countries whose wealth is sensitive to world stock market dynamics, the resulting systematic risk requires a currency risk premium that makes the carry trade profitable.

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Introduction

The uncovered interest rate parity hypothesis maintains that gains from trading on the interest rate differential between two countries—borrowing in the low-interest currency and investing into the high-interest currency—should be eliminated by a commensurate depreciation of the high interest rate currency. Empirically, this hypothesis has been shown to be violated for a number of currencies over sustained periods of time. The most salient examples are the Japanese Yen and the Australian Dollar. Vis-a-vis the U.S. Dollar, the former boasts interest rates that are on average approximately 200bps below U.S. rates, while the latter country's rates are higher than U.S. rates by a similar order of magnitude. Notably, the Japanese Yen has tended to depreciate, while the Australian Dollar has systematically appreciated during this time. I develop a two-country dynamic asset pricing model to study the relationship between international bond markets and exchange rates, so as to understand whether these apparent anomalies are consistent with rational investors' demand to be compensated for risk.

The main results are as follows. While covered interest rate parity holds, uncovered interest rate parity (UIP) is violated, such that the carry trade—borrowing in the low-interest-rate currency and investing into the high-interest-rate currency—can be profitable in equilibrium. However, the risk, for which this profit is fair compensation, must have the 'right' sign. Carry trade will seem profitable when the curency of the high interest rate country features a positive risk premium. This implies that the country is characterized by strong production growth that can keep up with expected demand growth (leading to high interest rates), and simultaneously, total demand is dependent on exports to large countries, whose wealth is significanty exposed to the world stock markets. This dependency introduces systematic risk into demand dynamics, making it risky to hold that country's currency. This currency risk premium explicitly does not rely on market segmentation, i.e. a single stochastic discount factor prices all assets, but can be decomposed into a local market risk component and an exchange rate volatility component. Although the risk premium is not the result of skewness in the exchange rate distribution, the volatility of the exchange rate is shown to covary with the exchange rate itself over time, in a way that would lead to finding skewness in the time series. This is consistent with Brunnermeier et al. (2008), Jurek (2009) and Burnside et al. (2010).

I model a dynamic endowment economy with two countries, each producing a distinct good that can be traded cross-border by two representative investors. The two investors hold different beliefs about growth rates in both countries. The differences in beliefs, which may vary over time, create demand for risk sharing. This way of modeling heterogeneity captures differences in investors' assessment of the investment opportunity set or in their willingness to carry risk while retaining tractability. Financial markets, consisting of stock and bond markets in both countries, are complete and allow investors to share risk by trading internationally.

Both goods and financial markets are fully integrated. This is a critical difference to much of the existing literature of exchange rate models, where goods markets are implicitly segmented: in those models, interest rate differentials across countries arise from country-specific stochastic discount factors. A countries' production risk then becomes inextricably linked to the discount factor of the locally resident investor and exchange rate dynamics and risk premia are largely driven by the market segmentation and reflect its cost to investors.

In this model, as in single-good economies, interest rates reflect investors' expectations about consumption growth rates and the associated risk. Since total consumption demand in a two-good economy is split across the two goods, the dynamics of a good's supply relative to demand for it become important for consumption growth. While the supply in a Lucas economy remains exogenous, total demand for goods is determined endogenously by the changes in investors' wealth and their expectations about the future. The implications of this are as follows.

First, while the interest rate differential satisfies covered interest rate parity, uncovered interest rate parity is violated in equilibrium. Carry trade profits reflect compensation for exchange rate risk: if holding a country's currency is a bad hedge against systematic risk, its return must be high, the currency appreciates on average.

The interest rate differential across the two countries reflects differences in growth expectations: the country in which demand growth is expected to be more easily served by a sufficiently strong growth in output has the higher interest rate. If, however, relative demand of that country's good is also volatile due to being dependent on exports, the currency is expected to appreciate—UIP will be violated. The currency appreciation is compensation for systematic risk: demand is endogenous, determined by how aggregate risk is allocated across investors, and thus covaries with systematic risk.

The conditions for a typical carry trade situation require the high interest rate country to have systematically risky demand. This is likely to occur when this country's trade partners are large countries whose investors carry large amounts of the world's aggregate risk through their investment portfolio.

This suggests that one should find profitable carry trade in situations where the high interest rate country features strong growth on the production side that can in expectation keep up with future demand growth, but where demand for its good is erratic relative to supply, due to being dependent on exports to a country whose wealth is exposed to stock market risk. This is consistent with Jylha, Suominen, and Lyytinen (2008) and Ranaldo and Soderlind (2010), who find that carry returns are positively correlated with the risk premium on equity.

Second, although instantaneous returns are Gaussian—and thus exhibit no skewness per se—equilibrium parameters vary over time, which can give the impression of skewness in the time series. For a carry trade transaction, skewness is detrimental if the volatility of the exchange rate rises just as the exchange rate moves against the carry trade, i.e. negative skewness. In the model, the covariance between the exchange rate and its volatility is negative if aggregate risk is unevenly distributed across the two agents. The same conditions that make a profitable carry trade relationship between two countries more likely, also generate exchange rate dynamics that are consistent with finding a negatively skewed distribution in the data. So, while this model does not propose a 'skewness premium', its implications are consistent with finding a relationship between carry trade and skewness characteristics of currencies—albeit a correlation, not a causation.

Third, introducing a constraint into the model as an extension allows me to address how sudden funding constraints impact the currency market and whether they harm carry trade positions. To the extent that limiting an investor's leverage reduces the disparity in risk sharing across investors, demand for goods becomes more stable and less sensitive to market shocks, thus lowering the risk premium on exchange rates. This drop in expected appreciation of the high interest rate currency means the carry trade becomes less profitable.

The present model builds on the theoretical literature looking at international financial markets. While this literature is vast, this paper is most closely related to recent models of international financial markets, for example Zapatero (1995), who studied exchange rates in a two-good world and how they covary with equity markets. He also shows the conditions necessary for international markets to complete local, independently incomplete, stock markets. Bansal and Shaliastovich (2007) model currency and bond markets driven by local inflation and strict segmentation of goods markets along country borders. In contrast, I allow for open goods markets as well as financial markets in establishing equilibrium exchange rates. Verdelhan (2010) proposes a habit-based explanation for UIP violation, where interest rate differentials are determined by investors' different risk aversions. In contrast to the type of heterogeneity assumed in the paper here, the habit formation setup of Verdelhan (2010) requires that an investor's risk aversion, and the resulting consumption-savings decision, affects exclusively the investor's local interest rates; as in Bansal and Shaliastovich (2007), consumption markets are de facto completely segmented. Colacito and Croce (2011) study the equity premium in an international context and find a link between long-run growth and exchange rates, likewise in a setup with strictly segmented goods markets. The model in this paper aims to avoid full segmentation of goods markets, in order to provide hypotheses that are testable in open economies where trade in goods markets is not negligible. I do however allow for heterogeneity across agents, in the form of differences in beliefs, as analyzed in detail in Basak (2000), among others.

In my model, all investors have, a priori, access to all markets. Assuming individual country bonds permits an analysis of interest rate differentials.[1] My setup is close to that of Pavlova and Rigobon (2008), though their focus is on the correlation of international stock markets.

[1]Though Barr and Priestley (2004) find that international bond markets are not fully integrated, with local market risk having significant impact on returns, Warnock and Warnock (2009) show that international capital flowing into the US government bond markets has contributed to lowering Treasury yields. This suggests that foreign investors' consumption-savings decision do affect local interest rates, which my model allows.

This analysis provides a possible explanation for the frequently observed violation of uncovered interest rate parity. It can also reconcile two strands of the empirical literature on currency risk premia. Consistent with economic intuition that only the systematic component of exchange rate volatility should be priced, Lustig et al. (2009) empirically link carry trade returns to global risk factors. But others, e.g. Menkhoff et al. (2010), have found overall exchange rate variance to have explanatory power for currency returns. In the model proposed in this paper, the currency risk premium can be decomposed into two elements that contribute to global consumption growth risk: the risk premium for local production risk and the exchange rate's variance.

Giving investors access to a broad set of financial markets as well as free trade in goods helps provide more precise predictions on the conditions under which currency risk premia arise in open economies, relating them to countries' wealth and the role of import and export to their local economy.

The paper proceeds as follows. Section 1 introduces the model. Section 2 derives the equilibrium. Section 3 studies interest rates and their relationship to exchange rates. Section 4 concludes. Proofs are in the Appendix.

I Model

I.A The Economy and Investor Preferences

The pure exchange economy is comprised of two countries, *home* and *foreign*, each of which specializes in the production of one good, $j = h, f$. While production is specialized, consumption is not: the two representative investors, $i = H, F$, that respectively populate the two countries derive utility from the consumption of both goods. To this end, the goods markets are frictionless, meaning there are no transportation costs or tariffs, such that both investors face the same relative price for the two goods. The output processes of the two goods are given by

$$dY_t^h = \mu_{Y_h} Y_t^h dt + \sigma_{Y_h} Y_t^h dW_{t,\mathsf{h}},$$
$$dY_t^f = \mu_{Y_f} Y_t^f dt + \sigma_{Y_f} Y_t^f dW_{t,\mathsf{f}}. \quad (1)$$

The two countries have potentially different growth rates μ_{Y_h} and μ_{Y_f}, and the production technologies are subject to uncorrelated shocks: σ_{Y_h} and σ_{Y_f} characterize the sensitivity of output to these fundamental shocks.[2] The uncorrelated Brownian motions $dW_{t,\mathsf{h}}$ and $dW_{t,\mathsf{f}}$ represent the *home* and *foreign* countries' respectively local production shocks.

Investor i maximizes expected utility $E\left[\int_0^T u_i\left(C_{it}^{\mathsf{h}}, C_{it}^{\mathsf{f}}\right) dt\right]$, subject to his budget constraint and potentially binding additional allocation constraints. Utility functions of both investors are separable and additive over the two goods in the economy, but investors have a local bias in their consumption preferences: the *home* investor H weights his local good Y^h more highly in his utility, while the *foreign* investor F prefers his own local good Y^f.

$$u_H\left(C_{Ht}^{\mathsf{h}}, C_{Ht}^{\mathsf{f}}\right) = \alpha_t^H \log C_{Ht}^{\mathsf{h}} + (1-\alpha_t^H)\log C_{Ht}^{\mathsf{f}}, \quad (2)$$
$$u_F\left(C_{Ft}^{\mathsf{h}}, C_{Ft}^{\mathsf{f}}\right) = (1-\alpha^F)\log C_{Ft}^{\mathsf{h}} + \alpha^F \log C_{Ft}^{\mathsf{f}}. \quad (3)$$

The preference parameters α_t^H and α^F are assumed to be between 0.5 and 1, capturing the home bias in consumption. Time variation in this relative preference is driven by demand shocks, which follow a martingale uncorrelated with production shocks:

$$d\alpha_t^H = \sigma_{t,\alpha} dW_{t,\alpha}. \quad (4)$$

To ensure that α_t^H remains above 0.5, $\sigma_{t,\alpha}$ must vary over time. For example, H's preferences may be related to an underlying state variable x_t taking the form $\alpha_t^H = 1 - 0.5/(1+x_t)$.[3] A home bias

[2]Time-subscripts on parameters μ and σ above are suppressed, in the interest of parsimony of notation. As a special case, the production processes can be assumed to follow geometric Brownian motions, but the model goes through as long as the parameters are assumed to be adapted processes.

[3]This ensures α_t^H remains within the appropriate bounds if x_t follows an Ito process; details can be found in the

in consumption patterns is consistently found empirically, often attributed to the non-tradability of certain goods (notably services) and familiarity. For parsimony, in this model the bias and its time variation is not modeled in detail but exogenously imposed in (4).[4]

I.B Financial Markets

The capital markets consist of two positive net supply stocks as well as local zero net supply bonds, which are accessible to both investors. The risky claim to the future output of *home* good Y^h will be referred to as the *home stock* stock S_t^h, and S_t^f is the claim to future output of *foreign* good Y^f. The place of listing is immaterial—there are no differential transaction costs of trading stocks for the two agents. The 'geography' of the stocks is determined simply by the good they are a claim to. The two stocks follow the dynamics

$$dS_t^h = \mu_t^{S_h} S_t^h dt + \vec{\sigma}_t^{S_h} S_t^h d\vec{W}_t, \tag{5}$$

$$dS_t^f = \mu_t^{S_f} S_t^f dt + \vec{\sigma}_t^{S_f} S_t^f d\vec{W}_t, \tag{6}$$

where expected return and volatility parameters $\mu_t^{S_j}$ and $\vec{\sigma}_t^{S_j}$ are determined in equilibrium. $\vec{\sigma}_t^{S_j}$ is the three-dimensional vector of stock S_t^j's sensitivities with respect to the mutually uncorrelated supply and demand shocks $d\vec{W}_t = \left(dW_{t,\mathsf{h}}, dW_{t,\mathsf{f}}, dW_{t,\alpha}\right)^\top$.

Bonds are traded in both countries, creating 'locally' riskless assets that effectively provide a forward contract on one unit of future local production.

$$db_t^h = r_t^h b_t^h dt \qquad \text{in terms of good } Y_t^h, \tag{7}$$

$$db_t^f = r_t^f b_t^f dt \qquad \text{in terms of good } Y_t^f. \tag{8}$$

While default is ruled out, the real exchange rate between *home* and *foreign* countries makes bond

appendix. Another example of an admissible process is $\alpha_t^H = E[\alpha_T^H | \mathcal{F}_t]$, where the terminal value of the preference parameter is a random variable between 0.5 and 1.

[4]This approach to modeling demand shocks is consistent with Dornbusch, Fischer, and Samuelson (1977), who noted the importance of allowing for demand shifts in an international model of multiple-good economies. The particular form of demand shocks has been used previously, e.g. in Pavlova and Rigobon (2008) and Schornick (2009).

payoffs potentially risky in terms of consumption choices. The relative value of the two goods to the investors varies over time, as it responds to changes in preferences and output. Equilibrium goods prices p_t^j and their relative price $\bar{p}_t = p_t^h / p_t^f$ are determined by supply of and demand for each of the goods at time t.

These 'terms-of-trade' function as a real exchange rate between the countries, imposing potential exchange rate risk on the securities. After converting asset prices into units of the numeraire good, bond prices follow $dB_t^j = d(p_t^j b_t^j)$. As such, countries' bond yields reflect the state of the local economy in relation to the world economy.

Without loss of generality, the *foreign* good is set as the numeraire good in the remainder of the paper, rendering $p_t^f = 1$ and thus B_t^f the instantaneously riskless asset in this economy.

I.C Information Structure

Uncertainty in the economy is characterized by the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$. However, investors hold different beliefs about the expected growth rates of the two economies, *home* and *foreign*. Even receiving identical new information on economic fundamentals, investors continue to rationally disagree with one another if their beliefs are based on different initial priors regarding the two growth rates.[5] Investors base their decision on the information given by observing economic output and demand shocks; the incomplete filtration $\{\mathcal{F}_t^{Y_{i,j}}\}$ is generated by processes Y_t^h and Y_t^f.

The relation between the two rational investors' beliefs is determined by observational equivalence.

$$\begin{aligned} dY_t^j &= \mu_{Y_j} Y_t^j dt + \sigma_{Y_j} Y_t^j dW_{t,\text{j}} \\ &= m_{Y_j,t}^{\text{(H)}} Y_t^j dt + \sigma_{Y_j} Y_t^j dW_{t,\text{j}}^{\text{(H)}} \\ &= m_{Y_j,t}^{\text{(F)}} Y_t^j dt + \sigma_{Y_j} Y_t^j dW_{t,\text{j}}^{\text{(F)}}. \quad \text{for j= h,f} \end{aligned} \tag{9}$$

Investors H and F attribute different portions of the observed output increase dY_t^h to expected

[5]The volatility components of economic output are known by investors. Quadratic variation allows them to draw exact inferences about the diffusion terms of dY_t^h and dY_t^f, as well as demand shocks $d\alpha_t^H$. This form of 'agreeing to disagree' was noted by Morris (1995) and has been widely used in the asset pricing literature. See, e.g. Basak (2005).

growth, $m^{(\mathsf{H})}_{Y_h,t}$ and $m^{(\mathsf{F})}_{Y_h,t}$, and their consumption and investment decisions will reflect these perceptions. The relationship between investors' perceptions about the three uncorrelated sources of risk in the economy is given by

$$d\vec{W}^{(\mathsf{F})}_t \quad = \quad d\vec{W}^{(\mathsf{H})}_t - \Delta\vec{m}_{t,Y}\,dt \qquad \text{where} \qquad \Delta\vec{m}_{t,Y} = \vec{\Sigma}^{-1}(\vec{m}^{(\mathsf{F})}_t - \vec{m}^{(\mathsf{H})}_t) \tag{10}$$

where Σ is the 3×3 diffusion matrix of the economy's fundamental processes, output and demand:

$$\begin{pmatrix} dW^{(\mathsf{F})}_{t,\mathsf{h}} \\ dW^{(\mathsf{F})}_{t,\mathsf{f}} \\ dW^{(\mathsf{F})}_{t,\alpha} \end{pmatrix} = \begin{pmatrix} dW^{(\mathsf{H})}_{t,\mathsf{h}} \\ dW^{(\mathsf{H})}_{t,\mathsf{f}} \\ dW^{(\mathsf{H})}_{t,\alpha} \end{pmatrix} - \begin{pmatrix} \sigma_{Y_h} & 0 & 0 \\ 0 & \sigma_{Y_f} & 0 \\ 0 & 0 & \sigma_{t,\alpha} \end{pmatrix}^{-1} \begin{pmatrix} m^{(\mathsf{F})}_{Y_h,t} - m^{(\mathsf{H})}_{Y_h,t} \\ m^{(\mathsf{F})}_{Y_f,t} - m^{(\mathsf{H})}_{Y_f,t} \\ 0 \end{pmatrix} dt. \tag{11}$$

The elements of $\Delta\vec{m}_{t,Y}$ capture the relative optimism of investor F compared to investor H regarding the two countries' growth rates. As demand shocks follow a martingale, there is no room for rational disagreement, the last element of $\Delta\vec{m}_{t,Y}$ is equal to zero. A 'home bias' about investment opportunities would be captured by a negative first element of $\Delta\vec{m}_{t,Y}$, and a second positive element.[6]

Although not modeled in detail in this paper, the time-subscripts in $m^{(i)}_{Y_h,t}$ and $m^{(i)}_{Y_f,t}$ reflect that beliefs could be subject to learning, as investors update their beliefs using observed output growth and potentially other observable processes as signals. Specific assumptions about how investors learn over time are not critical to establishing equilibrium in this model, as long as the process of investor disagreement can be assumed to be bounded.

[6]The foundations for the assumption of different priors have been discussed for the general case in Morris (1994), and similar setups in single-good economies can be found, e.g. in Basak (2000) who includes extraneous risk, and Gallmeyer and Hollifield (2008).

II Equilibrium

I establish equilibrium by aggregating both investors with their appropriate weight into a representative agent.

$$U(C_H, C_F) = u_H\left(C^h_{Ht}, C^f_{Ht}\right) + \lambda_t u_F\left(C^h_{Ft}, C^f_{Ft}\right), \tag{12}$$

where the weight of investor H is normalized to 1, thus λ_t captures the weight of investor F relative to investor H that results from competitive equilibrium. This relative weight reflects initial endowments and differences in investors' state price densities—to the degree that investors' different assessment of economic fundamentals leads them to assign different values to possible future states: $\lambda_t = \psi_H \xi^H_t / \psi_F \xi^F_t$. Both investors' budget constraints

$$dX^i_t = X^i_t\left[\sum_{j=h}^{f} \pi^{S_j}_{it}(dS^j_t + p^j_t Y^j_t dt)/S^j_t + \sum_{j=h}^{f} \pi^{B_j}_{it} dB^j_t / B^j_t\right] - \sum_{j=h}^{f} p^j_t C^j_{it} dt \qquad \text{for } i = H, F \tag{13}$$

where $X^i_t \geq 0$ is agent i's wealth, $\pi^{S_j}_{it}$ is the wealth fraction investor i chooses to invest in stock S^j_t, and $\pi^{B_j}_{it}$ the fraction invested into bond B^j_t, must be satisfied in equilibrium.

Under belief heterogeneity, even a complete market will reflect differences across agents' state price densities, due to differences in perception that are then reflected in portfolio holdings and market prices of assets. While such differences are not necessary for an equilibrium with exchange rate risk premia to emerge in this setting, allowing for investor heterogneity can help align the model with empirical finding, many of which involve market reactions to investors' apparent changes in beliefs. Using investors' beliefs as defined in (10), state price densities follow

$$d\xi^H_t = -r_t \xi^H_t dt - \vec{\kappa}^{H^\top}_t \xi^H_t d\vec{W}^{(H)}_t, \tag{14}$$

$$d\xi^F_t = -r_t \xi^F_t dt - \vec{\kappa}^{F^\top}_t \xi^F_t d\vec{W}^{(F)}_t. \tag{15}$$

where $\vec{\kappa}^i_t = \vec{\sigma}^{-1}_{S,t}\left(\vec{m}^{(i)}_{S,t} - r_t \mathbf{1}\right)$ is investor i's market price of risk based on his beliefs about both countries' fundamental growth rates, and by observational equivalence $\Delta\vec{\kappa}_t = \Delta\vec{m}_{t,Y}$ from (10) must hold.

Proposition 1 gives equilibrium consumption, portfolio holdings and stock prices. Though production risk of the *home* and *foreign* economies is uncorrelated, stock and bond markets are related through equilibrium prices of the countries' output, arising from investors' consumption and investment choices. The equilibrium terms of trade $\bar{p}_t = p_t^h / p_t^f$ reflect relative demand for and supply of countries' output. It determines the real exchange rate between the countries and is the conductor for fundamental shocks to propagate from the goods into the financial markets.

Proposition 1. *Market-clearing consumption shares of good $i = h, f$ are characterized by s_i^F for agent F, and $(1 - s_i^F)$ for investor H.*

$$\begin{aligned} C_{Ft}^h &= \frac{\lambda_t \left(1 - \alpha^F\right)}{\alpha_t^H + \left(1 - \alpha^F\right) \lambda_t} Y_t^h = s_h^F Y_t^h; \qquad C_{Ht}^h = (1 - s_h^F) Y_t^h \\ C_{Ft}^f &= \frac{\lambda_t \alpha^F}{1 - \alpha_t^H + \alpha^F \lambda_t} Y_t^f = s_f^F Y_t^f; \qquad C_{Ht}^f = (1 - s_f^F) Y_t^f \end{aligned} \tag{16}$$

Taking good Y_t^f to be the numeraire, equilibrium stock and bond prices in the home *country are functions of the relative price of the local good, $\bar{p}_t = p_t^h / p_t^f = \xi_t^h / \xi_t^f$.*

$$S_t^h = \bar{p}_t Y_t^h (T - t), \tag{17}$$

$$B_t^h = \bar{p}_t b_t^h, \tag{18}$$

$$S_t^f = Y_t^f (T - t), \tag{19}$$

$$B_t^f = b_t^f \qquad \forall t \tag{20}$$

$$where \quad \bar{p}_t = \frac{\alpha_t^H + (1 - \alpha^F) \lambda_t}{(1 - \alpha_t^H) + \alpha^F \lambda_t} \frac{Y_t^f}{Y_t^h}. \tag{21}$$

λ_t follows dynamics $d\lambda_t = \lambda_t \Delta \vec{\kappa}_t^\top d\vec{W}_t^{(H)}$, where $\Delta \vec{\kappa}_t^\top = \left[\Delta m_t^h, \Delta m_t^f, 0\right]$ capture differences in investors' market prices of home, foreign, and demand risk. Portfolio weights of investors $i = H, F$ are

$$\pi_{it} = (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \left(\vec{m}_{S,t}^{(i)} - r_t \mathbf{1}\right) \tag{22}$$

where $\pi_{i,t} = [\pi_{i,t}^{S_h}, \pi_{i,t}^{S_f}, \pi_{i,t}^{B_h}]^\top$ are the fractions of i's wealth invested into the two stocks and the home *bond. The budget constraint implies $\pi_{i,t}^{B_f} = 1 - \mathbf{1}^\top \pi_{i,t}$.*

III Interest Rates and Exchange Rate Risk Premia

The relationship between the equilibrium interest rates of the two countries allows us to assess the exchange rate risk premium implied by the model. While Covered Interest Parity is ensured to hold by arbitrage in this complete-market model, Uncovered Interest Parity (UIP) generally fails. The failure of UIP gives rise to the so-called 'Carry Trade' under certain conditions. The large literature devoted to the carry trade or 'forward premium puzzle' began with the seminal paper by Fama (1984).

III.A The Riskfree Interest Rates in Home and Foreign Country

Equilibrium interest rates are determined by investors' consumption and savings decisions, reflecting their expectations of consumption growth rates and the volatility of their future consumption. Although there are two default-free securities available to investors, in terms of the numeraire good there is only one 'riskfree' asset: the *foreign* bond B_t^f, guaranteeing one future unit of the numeraire (*foreign*) good.[7] While this does not provide a riskfree unit of either agent's 'consumption basket' (which is always comprised of both goods), it is similar to the type of riskfree asset provided by the existing government bond markets, which are likewise based on an exogenously chosen basket. More critically, the results of this paper do not depend on the choice of numeraire good.

In equilibrium, the riskfree rate reflects market views on expected consumption growth rates

[7] Proposition 1 gives the results taking the *foreign* good Y_t^f to be the numeraire, though this is without loss of generality. Either of the two goods or a combination thereof can be used as numeraire good. In particular, the *relative* pricing of assets remains identical. A consumption basket composed of fraction β of the *home* good Y_t^h and fraction $(1-\beta)$ of Y_t^f implies that relative goods prices are defined as $\beta p_t^h + (1-\beta) p_t^f = 1$.

and the associated risks.

$$r_t^f = \mu_{C_{Hf}} - \sigma_{C_{Hf}}^2 f - \frac{\sigma_{C_{Hf}}\sigma_{\alpha_t}}{1-\alpha_t^H} \quad (23)$$

$$= \mu_{C_{Hh}} + \mu_{p^h} - \sigma_{p^h}^2 - \sigma_{C_{Hh}}^2 - \mathsf{cov}_{C_{Hh},p^h} + \frac{1}{\alpha_t^H}\mathsf{cov}_{\alpha,p^h} + \frac{1}{\alpha_t^H}\mathsf{cov}_{C_{Hh},\alpha} \quad (24)$$

It is possible to express interest rates in terms of consumption growth in the asset local to the interest rate in question, or in terms of consumption growth of the other good. While identical, the former is easier to interpret as it circumvents translation into the numeraire good. Rewriting consumption growth into economic fundamentals of the model gives

$$r_t^f = s_f^F m_{Yf,t}^{(F)} + (1-s_f^F)m_{Yf,t}^{(H)} - \sigma_{Yf,t}^2, \quad (25)$$

$$r_t^h = s_h^F m_{Yy,t}^{(F)} + (1-s_h^F)m_{Yh,t}^{(H)} - \sigma_{Yh,t}^2, \quad (26)$$

where $s_f^F = \frac{\lambda_t\alpha^F}{1-\alpha_t^H+\alpha^F\lambda_t}$ is F's consumption share of total consumption in *foreign* good Y_t^f, and $s_h^F = \frac{(1-\alpha^F)\lambda_t}{\alpha_t^H+(1-\alpha^F)\lambda_t}$ is F's consumption share of total consumption in *home* good Y_t^h, as given in (16). Both rates r_t^f and r_t^h, are positively related to aggregate consumption growth in their respectively local good, and negatively related to the associated aggregate consumption risk, due to the precautionary savings motive of risk-averse agents.

Under heterogeneity, rates are determined by the weighted average of investors' beliefs about aggregate consumption growth rate. This average is higher than the expectation of a benchmark representative agent economy, i.e. in the absence of disagreement. Using as an example the *foreign* rate, it can be shown that $E^H[dC_{Hf,t}] + E^F[dC_{Ff,t}] = \mu_{Y_f} + s_f^F(1-s_f^F)(\Delta m_t^{Y_h\,2} + \Delta m_t^{Y_f\,2})$. Since investors agree to disagree, both believe themselves to be making superior consumption and investment decisions, and thus assume they will have higher consumption growth than the other in the future. This results in upwards pressure on r_t^f, which, due to log utility of both agents is offset by the disagreement risk the belief heterogeneity presents.[8]While (25) indicates that investors' expec-

[8]It can be shown that interest rates load more strongly on consumption growth risk in high interest rate countries than they do in low-interest rate countries. This is consistent with Lustig and Verdelhan (2007).

tations about fundamental *home* growth rates do not impact r_t^f directly, there is an indirect impact. The dynamics of the rate depend on investors' perception about *home* fundamentals through the endogenous dynamics of λ_t. Beliefs about relative fundamentals in both economies are reflected in investors' state price densities, which determine investment decisions and thus equilibrium rates. The analogous effect applies to the *home* rate r_t^h in (26).[9]

III.B Carry Trade Returns and Negative Skewness

In an economy with floating real exchange rates, the relationship between two countries' interest rates is determined by the exchange rate in an arbitrage relationship, Covered Interest Parity. One investor's state price density—his assessment and valuation of possible future states—can be denominated in terms of either good, *home* or *foreign*. Where $\xi_t^H = \lambda_t \xi_t^F$ relates the two investors' state price densities to one another in terms of the same good, the terms of trade $\bar{p}_t$ translates one investor's state price density from denomination in terms of one good into the other.: $\xi_t^h = \bar{p}_t \xi_t^f$.[10] This is a critical difference to models with segmented goods markets. There, the production risk of one country becomes inextricably linked to the state price density of the locally resident investor. This effectively makes the exchange rate, or terms of trade, function as a measure of investors' heterogeneity, reflecting the 'price' of goods market segmentation.

The rate earned on the *home* country's bond, as given in (26) is risky in terms of the numeraire Y_t^f. Adjusting for the riskiness of the price in terms of the numeraire gives $dB_t^h = d(p_t^h b_t^h)$.[11]

The theory of uncovered interest rate parity (UIP) posits that interest rates should be related by the expected appreciation among the two currencies, suggesting that currencies of countries with higher interest rates will depreciate over time, to compensate for the higher return investors can

[9] Recall however, that this rate r_t^h as expressed above denoted in terms of the *home* currency, not in terms of the designated numeraire Y_t^f.

[10] As one example in the literature, see Colacito and Croce (2011).

[11] The exchange rate is often defined as demand-weighted goods prices, according to $\left(\frac{p_t^h}{\alpha_t^H}\right)^{\alpha_t^H} \cdot \left(\frac{p_t^f}{1-\alpha_t^H}\right)^{1-\alpha_t^H} / \left(\frac{p_t^h}{1-\alpha^F}\right)^{1-\alpha^F} \cdot \left(\frac{p_t^f}{\alpha^F}\right)^{\alpha^F}$. While p_t^h rather reflects the terms of trade, in this specialized economy where each country produces exclusively one good, $\bar{p}_t$ is the exchange rate that determines Covered Interest Parity. In any case, the two definitions are positively correlated.

earn in the high-interest rate currency.

Empirical evidence however shows that carry trade transactions have been profitable for significant periods of time: the profits of borrowing in a low-interest-rate currency and investing it into a high-interest-rate currency is not only not canceled out by depreciation of the high-interest rate currency, it is indeed often the case that this currency instead appreciates.

Covered Interest Parity—the relationship between *home* and *foreign* interest rates enforced by arbitrage—necessarily reflects any risk premia , and thus permits an assessment of when such risk premia lead to profitable carry trades and can explain this apparent violation of UIP.

CIP reflects the optimality of investors' consumption decisions: the marginal utility of consuming *foreign* goods is equal to the marginal utility of consuming *home* goods, scaled by the prevailing exchange rate: $\xi_t^{Y_f} = \xi_t^{Y_h}/p_t^h$. Therefore, the following relationship must hold at all times t:

$$r_t^h = r_t^f - \mu_{p_h,t} + \sigma_{p_h,t}\kappa_t \tag{27}$$

where $\mu_{p_h,t}$ and $\sigma_{p_h,t}$ are the drift and diffusion of p_t^h, the expected appreciation of the *home* currency and its volatility.[12] The interest rate offered on *home* bonds (in units of *home* 'currency', r_t^h, will be equal to r_t^f (the riskfree rate for this economy with numeraire Y_t^f) less the expected appreciation of the *home* currency, plus the risk premium for taking on exchange rate risk.

Uncovered Interest Parity relies on the forward rate being an unbiased predictor of future spot rates, which inherently implies risk-neutrality or exchange rates that are orthogonal to systematic risk. (27) is consistent with this: if the market price of risk κ_t were zero as in a risk-neutral world or exchange rates are orthogonal to priced risk, the last term in (27) would be zero. Accordingly, only expected exchange rate movements $\mu_{p_h,t}$ would determine rate differentials. Interest rates in the *home* country would then be higher than r_t^f only if $\mu_{p_h,t} < 0$, i.e. the price of the home good is expected to fall—the high interest rate currency depreciates.

[12]Note that the terms herein are necessarily from the viewpoint of one investor, H or F. As they disagree on growth rates, they will likewise disagree on the market price of risk κ_t as well as the expected exchange rate appreciation $\mu_{p_h,t}$. This arbitrage relationship will hold from either investor's view, however, when all terms are appropriately adjusted for the disagreement.

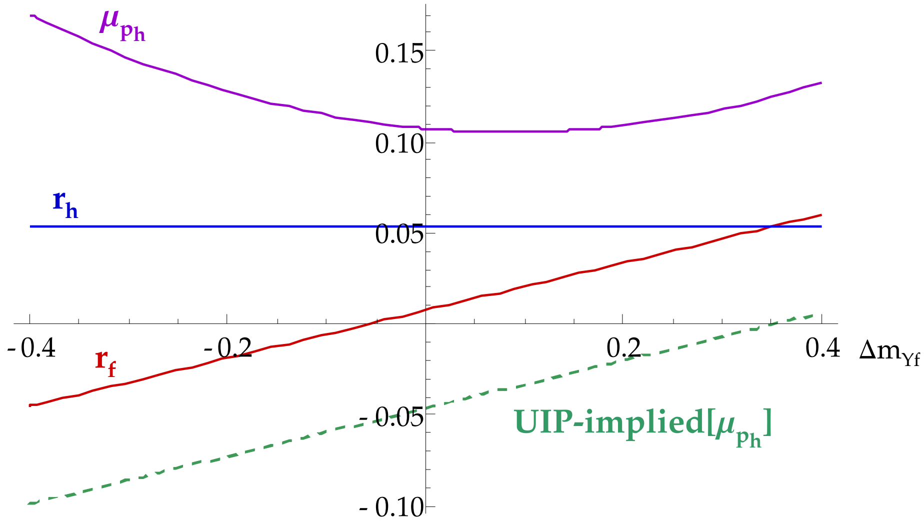


Figure 1: **UIP-Implied Depreciation** If UIP held, $r_t^f - r_t^h$ should be the expected appreciation of the *home* currency. This $\mu_{p_h,t}$ (dashed) implied by UIP theory is negative, in contrast to the true value (purple). The difference is the exchange rate risk premium.

Fig. (1) is an illustration of scenarios where a positive interest rate differential $r_t^h - r_t^f$ would, under UIP, suggest that a depreciation of the *home* currency is expected (dashed line), but the presence of the risk premium leads to a positive rate differential despite expected currency appreciation, $\mu_{p_h,t} > 0$.

In equilibrium investors are compensated only the carrying systematic risk as characterized by $\sigma_{p_h,t}\kappa_t$, the exchange rate's covariance with the marginal utility of consumption. Not exchange rate variation itself, but only the component of exchange rate variation that is not orthogonal to priced risk matters. However, empirically exchange rate variation itself has been shown to matter. This finding is consistent with (27). κ_t is the market price of risk in terms of the numeraire good. Rewriting this in terms of price of local (*home*) risk gives

$$r_t^h = r_t^f - \mu_{p_h,t} + \sigma_{p_h,t}^2 + \sigma_{p_h,t}\kappa_t^{home}, \tag{28}$$

separating components of the 'world' market price of risk into local production risk and exchange rate risk.

Structurally, interest rates in this two-country/two-good world are similar to the rate in a single-country economy with only one good: high output growth rates imply high interest rates, high aggregate risk in the form of volatile output lowers interest rates.[13] When total consumption demand is split across two different goods, what becomes relevant for determining market prices of risk and interest rates is the growth of output *relative* to the growth of demand for that good. Equilibrium state price density in terms of *foreign* risk[14] $\xi^H_{f,t} = \frac{1-\alpha^H_t}{C^f_H \psi_H} = \frac{\alpha^H_t}{C^h_H \psi_H \bar{p}_t}$ depends on both demand and supply for the respective goods. While output is an exogenous process in a Lucas Tree economy, total demand depends on (time-varying) preferences α^H_t and α^F as well as the respective weight λ_t of the two investors in equilibrium. Hence, it is not simply countries' output growth and its associated risk that determines interest rates, but the growth of output relative to growth in demand, and how demand covaries with output, that denotes aggregate risk in this environment. Unsegmented goods markets are critical for this. If all local output is always consumed by local agents, demand is necessarily given by one agent, as in a benchmark representative environment. This will render what is here $\bar{p}_t$ and λ_t inseperable.

Defining total demand—across both investors H and F—for the two goods as $\mathcal{D}_{Y_h} = \alpha^H_t + (1 - \alpha^F)\lambda_t$ for the *home* good and $\mathcal{D}_{Y_f} = 1 - \alpha^H_t + \alpha^F \lambda_t$ for the *foreign* good, allows us to rewrite (27) using $\xi_{h,t} = \mathcal{D}_{Y_h,t}/Y^h_t$ and $\xi_{f,t} = \mathcal{D}_{Y_f,t}/Y^f_t$ as

$$r^h_t = r^f_t - \mu_{p_h,t} - \sigma_{\xi_f,t}\sigma_{p_h,t} \tag{29}$$

$$r^h_t = r^f_t - \left(\mu_{\mathcal{D}_h/Y_h} - \mu_{\mathcal{D}_f/Y_f} + \sigma^2_{\mathcal{D}_f/Y_f} - \sigma_{\mathcal{D}_h/Y_h}\sigma_{\mathcal{D}_f/Y_f}\right) - \sigma_{\mathcal{D}_f/Y_f}\left(\sigma_{\mathcal{D}_h/Y_h} - \sigma_{\mathcal{D}_f/Y_f}\right) \tag{30}$$

where the last term is the risk premium, $\sigma_{\xi_f,t}\sigma_{p_h,t}$.

There are two scenarios in which UIP can potentially be violated, and borrowing in the low-interest-rate currency to invest in the high-interest rate currency will have positive expected returns: first,

[13] For investors with non-logarithmic utility function, aggregate risk also includes disagreement risk when investors have heterogeneous beliefs. For a discussion of these factors, see e.g. Basak (2000).

[14] To reiterate, this is necessarily from one investor's perspective, in this case investor H. The same would hold when translating into investor F's perception. For parsimony of notation, the superscript 'H' has been suppressed but will be assumed unless explicitly mentioned.

conditions for a carry trade when $r_t^h > r_t^f$ and second, when $r_t^h < r_t^f$. The intuition for the conditions under which these interest rate differentials will violate UIP is the same in both cases, so only the first of these two cases is discussed here.

When $r_t^h > r_t^f$, UIP predicts $\mu_p < 0$. A violation of UIP occurs when instead $\mu_p > 0$, which implies $\sigma_{\xi_f,t}\sigma_{p_h,t} < 0$. The latter is the risk premium: a negative covariance between p_t^h and state price density ξ_t^f means the *home* currency is valuable in good times, thus holding the *home* currency is a bad hedge against systematic risk; thus, the return must be high, $\mu_p > 0$.

Regarding the interest rate differential, for r_t^h to be higher than its *foreign* counterpart r_t^f, the expected growth rate of '*home* demand relative to supply' must be lower than for the *foreign* country: the more investors expect future supply to be able to cover future demand for a country's good, the higher the interest rate will be in that country.[15] The risk premium is also related to the systematic risk of satisfying future demand with future supply.

$$\text{If } \sigma_{\xi_f,t}\sigma_{p_h,t} = \kappa_{f,t}\left(\kappa_{h,t} - \kappa_{f,t}\right) < 0 \text{ when } r_t^h > r_t^f \text{, UIP is violated.} \tag{31}$$

Recall that in this setup, the *foreign* good is the numeraire, therefore $\kappa_{f,t}$ can be seen as the 'world' market price of aggregate risk that prices all financial assets. Intuitively (as well as empirically), this should be positive, which means the differential in brackets in (31) must be negative: the demand relative to supply for the *home* good must be more volatile than that of the *foreign* country.

So while interest rates reflect expectations that demand will grow slowly relative to supply for the *home* good, the *home* currency is expected to appreciate—in violation of UIP—if the ratio of demand relative to supply is risky in the sense of exhibiting high volatility. From the definition of $\mathcal{D}_h/Y_h$ and $\mathcal{D}_f/Y_f$ this is more likely to happen if the *foreign* investor is rich (λ_t is high) and carries disproportionate amounts of aggregate risk ($\Delta\vec{m}_t$ is positive).[16] Since demand for goods is driven

[15]To ensure positive interest rates in a country, supply must be expected to grow faster than demand. The reverse can lead to negative rates.

[16]For a profitable carry trade when $r_t^f > r_t^h$ the intuition is analogous: the ratio of demand relative to supply for the *foreign* good is expected to grow more slowly, while this ratio being more volatile than the demand-supply ratio of *home* makes the *foreign* currency appreciate in expectation.

by preferences as well as endogenous factors like wealth distribution, the volatility of demand-to-supply will have systematic components, which are compensated in equilibrium and thus justify the higher interest rate.

The model suggests that one should empirically find a carry trade generate high returns where the high interest rate country has a growing economy on the production side that can, in expectation, keep up with future demand growth, but where demand is relatively erratic relative to supply, because the demand is dependent on exports to a rich country whose wealth is sensitive to stock market risk. This is consistent with Jylha, Suominen, and Lyytinen (2008) and Ranaldo and Soderlind (2010), who find that carry returns are positively correlated with the risk premium on equity.

In terms of economic fundamentals and endogenous state variable λ_t the risk premium $\kappa_t^f \sigma_{p_h,t}$ is

$$\kappa_t^f = \left[-s_f^F \Delta m_t^h \;,\; \sigma_{Y_f} - s_f^F \Delta m_t^f \;,\; \frac{1}{(1-\alpha_t^H + \lambda_t \alpha^F)} \sigma_{\alpha,t} \right] \tag{32}$$

$$\sigma_{p_h,t} = \left[-\frac{\lambda_t(\alpha^F - (1-\alpha_t^H))}{\mathcal{D}_{Y_h}\mathcal{D}_{Y_f}} \Delta m_t^h - \sigma_{Y_h}, -\frac{\lambda_t(\alpha^F - (1-\alpha_t^H))}{\mathcal{D}_{Y_h}\mathcal{D}_{Y_f}} \Delta m_t^f + \sigma_{Y_f}, \frac{1+\lambda_t}{\mathcal{D}_{Y_h}\mathcal{D}_{Y_f}} \sigma_{\alpha,t} \right] \tag{33}$$

where $\mathcal{D}_{Y_h}$ and $\mathcal{D}_{Y_f}$ is demand as defined above.

In the model, all returns are instantaneously Gaussian, so a skewness premium, which has been reported in the empirical literature, does not exist. Nonetheless, a time series of returns generated by this model could exhibit skewness, as the risk sensitivities of the exchange rate $\bar{p}_t$ is endogenous and thus time-varying. To coincide with what would appear to be a skewness premium, the skewness must be negative. In the scenario $r_t^h > r_t^f$ discussed here, skewness would be detrimental to the investor that sets up a carry trade position only if exchange rate volatility rises as the exchange rate itself falls. This type of 'skewness' would appear in a time series if $\text{Cov}_{\bar{p},vol(p)} < 0$. Using (33) we can show that this covariance is negative if aggregate risk is unevenly distributed across the two agents. In particular, if the *foreign* investor carries sufficiently high amounts of aggregate risk, in which case the carry trade is profitable in the above scenario. So, while 'skewness' is correlated with an exchange rate premium, the two are not causally related in this model.

III.C The Carry Trade Under Funding Constraints

One explanation often suggested for the failure of UIP is currency crash risk generated by carry trade investors experiencing financing constraints and having to withdraw suddenly from their positions. As these investors liquidate their positions, the currency falls, thus undoing any profit of the carry trade. As demonstrated above, a relatively simple Gaussian model without crash risk can generate currency risk premia that can potentially explain UIP. In this section the model is extended to consider how a financing constraint alters the equilibrium.

I impose a funding constraint in the form of an exogenously imposed limit on the leverage taken up by one of the investors, the *foreign* investor F. Such constraints are often the result of the difficulties in contingent contracting: bankruptcy costs and agency costs lead to limitations on leverage for most investors, either explicitly or implicitly through, e.g. margin constraints. In the aftermath of the 2007-2008 financial crisis and its repercussions around the world, the debate about leverage restrictions—and whether it must be in- or decreased—was reignited. Most investors engaged in the carry trade are institutional investors or money managers, who often face such constraints.Although the constraint analyzed here is exogenous, comparative statics on the constraint parameter can give some insight into how markets would react to tightening or loosening the level of leverage restrictions.

Utility functions and budget constraints remain as given in (2), (3) and (13).

While H is free to optimize his investment, F is limited in the amount of leverage he can take on by borrowing in bond markets: his positions in stocks cannot exceed a proportion $\eta > 1$ of his total wealth. The constraint can be expressed as

$$\mathtt{I}^\top \pi_{F,t} \leqslant \eta \; ; \qquad \mathtt{I} = [1,1,0]^\top \tag{34}$$

where $\pi_{F,t} = [\pi_{F,t}^{S_h}, \pi_{F,t}^{S_f}, \pi_{F,t}^{B_h}]^\top$ is the vector of F's portfolio holdings of investor in both stocks and the *home* bond.[17]

[17]Satisfaction of the budget constraint implies $\pi_{F,t}^{B_f} = 1 - \mathbf{1}^\top \pi_{F,t}$.

When the imposed constraint binds, optimal risk sharing is hindered by limiting the investor most willing to take on the risk from providing liquidity to the market. Prices in all financial markets have to accommodate this, and incentivize the other, unconstrained investor, to supply this missing liquidity. In order for such a constraint to lead to a significant rebalancing of portfolios, it must be that it is suddenly and unexpectedly imposed on the market.

The assumption of differences in beliefs is a technically tractable way to allow for constraints to bind with different degrees of severity—for a given level of leverage limitation η. How strict the constraint is, and how severely investors find themselves constrained by it, are two notions of a constraint's severity, but have a different impact on equilibrium. How the constrained investor adjusts his portfolio to compensate for the imposed restriction will depend on his beliefs about the alternative investment opportunities. If the binding limit on leverage η remains stable the same but the constrained investor's beliefs change, equilibrium market rates will change, despite the fact that overall leverage of the investor cannot change.

Constraints distort the desired portfolio choice: constrained investors seek alternative assets, in a manner that allows them to replicate their desired portfolio as closely as possible. The presence of constraints prevents investors from trading optimally, and the constrained investor must choose alternative investments that are permissible. How the optimal portfolio adjustments are determined, and how this distortion affects equilibrium, can be seen in the state price density ξ_t^F of investor F. This form allows us to distinguish the constrained investor's true assessment of investment opportunities from the density that is reflected by his actual portfolio choices. The distortions created in the equilibrium state price density by the constrained portfolio choices are captured by two parameters.[18] When the constraint binds, F's state price density changes from (15) to

$$d\xi_t^F = -\left(r_t + \delta(\upsilon_t)\right)\xi_t^F dt - \vec{\kappa}_{\upsilon,t}^{F^\top}\xi_t^F d\vec{W}_t^{(F)}. \tag{35}$$

where $\vec{\kappa}_{\upsilon,t}^F$ reflects F's true beliefs about the risk-return tradeoff as well as the restrictions the con-

[18] Cvitanic and Karatzas (1992) introduced the methodology to incorporate investment constraints on portfolio choice. Other related papers are e.g. He and Pearson (1991) and Cuoco (1997).

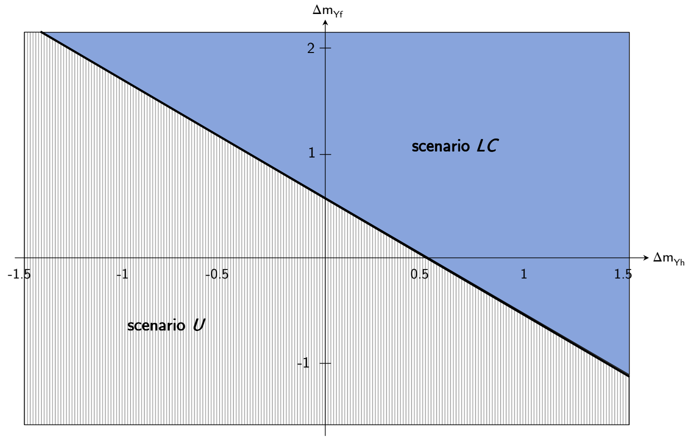


Figure 2: **Binding Leverage Constraint** The constraint binds ('scenario LC') if F is sufficiently optimistic about at least one of the two countries' investment opportunities. Strong optimism vis-a-vis one country is sufficient, as e.g. in the case of a 'home bias', in the second quadrant of the graph: $\Delta m_t^{Y_f} > 0$ and $\Delta m_t^{Y_h} < 0$.

straint places on his portfolio:

$$\vec{\kappa}_{\upsilon,t}^{F} = \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right) + \vec{\sigma}_{S,t}^{-1}\upsilon_t\mathtt{I}. \tag{36}$$

υ_t and $\delta(\upsilon_t)$ are scalar parameters that capture the effect of the leverage constraint on investor F's investment decisions: the constraint changes the relative attractiveness of all assets, including both bond markets, which both serve as a source of leverage for the investor.

Investors' beliefs about investment opportunities determine whether the constraint will bind for F: if he is sufficiently optimistic relative to investor H about economic growth rates in at least one of the two countries, the investment restriction will pose a problem. In order for markets to clear, prices must adjust. Fig. (2) shows the conditions under which the two constraints will, respectively, bind.

When the constraint binds, investor F's portfolio holdings appear inconsistent with the publicly observable riskfree interest rate. In contrast to the unconstrained investor H, his portfolio is distorted: he must reallocate the funds that he would like to invest into the stock markets to the bond markets or immediate consumption, making it appear as though he is making decisions based on different economic parameters, an adjusted interest rate $(r_t + \delta(\upsilon_t))$. The following proposition details the constrained equilibrium.

Proposition 2. *Still taking good Y_t^f to be the numeraire, equilibrium stock and bond prices retain the form detailed in* (20)*.*
$\bar{p}_t = p_t^h / p_t^f = \xi_t^h / \xi_t^f$ *holds, taking into consideration the changed dynamics of ξ_t^j , $j = h, f$. In particular,*

$$d\lambda_t = d\left(\frac{\psi_H \xi_t^H}{\psi_F \xi_t^F}\right) = \lambda_t \Delta\vec{\kappa}_t^\top d\vec{W}_t^{(H)} \tag{37}$$

where $\Delta\vec{\kappa}_t^\top = \left[\Delta\kappa_t^h, \Delta\kappa_t^f, \Delta\kappa_t^\alpha\right]$ capture differences in investors' market prices of home, foreign, and demand risk, which depends on the binding of the constraint: $\Delta\vec{\kappa}_t = \Delta\vec{m}_t^Y + \vec{\sigma}_{S,t}^{-1}(\upsilon_t \mathtt{I})$. Adjustment term υ_t is non-positive iff the constraint is binding, and zero otherwise. Portfolio weights of investors H and F are, respectively,

$$\pi_{Ht} = (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(H)} - r_t \mathbf{1}\right), \tag{38}$$

$$\pi_{Ft} = \begin{cases} (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) + (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \upsilon_t \mathtt{I} & \text{if } \upsilon_t < 0 \\ (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) & \text{otherwise} \end{cases} \tag{39}$$

where $\upsilon_t = \min\left(\frac{\eta - \mathtt{I}^\top (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right)}{\mathtt{I}^\top (\vec{\sigma}_{,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \mathtt{I}}, 0\right)$*:*

$$\upsilon_t = \begin{cases} \frac{-\sigma_{Yh}\sigma_{Yf}\sigma_\alpha^2\left[\Delta m_t^{Y_h}\sigma_{Yf} + \Delta m_t^{Y_f}\sigma_{Yh} - (\eta-1)(1+\lambda)\sigma_{Yh}\sigma_{Yf}\right]}{(\eta-1)^2\lambda_t^2(\alpha_t^H + \alpha^F - 1)^2\sigma_{Yh}^2\sigma_{Yf}^2 + (\sigma_{Yh}^2 + \sigma_{Yf}^2)\sigma_\alpha^2} & \text{if } \Delta m_t^{Y_h}\sigma_{Y_f} + \Delta m_t^{Y_f}\sigma_{Y_h} > (\eta - 1)(1 + \lambda_t)\sigma_{Y_f}\sigma_{Y_h}, \\ 0 & \text{otherwise.} \end{cases} \tag{40}$$

The collateral adjustment is $\delta(\upsilon_t) = -\eta\upsilon_t$.

$(\vec{\sigma}_{S,t}^{-1})^\top(\vec{\sigma}_{S,t}^{-1}\upsilon_t^{case}\mathtt{I}_{case})$ is the adjustment to F's portfolio in response to the binding leverage constraint. Having to reallocate his investment, F seeks assets—or portfolios thereof—that are highly correlated with the desired, but inaccessible investment. Thus, assets' covariance structure plays a key for the reallocation. The adjustment term υ_t captures the wedge that a binding constraint drives between F's true expectations about fundamental growth rates and the expectations reflected in asset prices via portfolio choice. The magnitude of this distortion to F's state price density depends on two characteristics: how strict the constraint is, i.e. the level of η, as well as how severely investor F is constrained, ie. the 'distance' between his desired position and the permissible one, conditional on a given η. The latter is determined by expectations about investment opportunities, $\Delta m_t^{Y_h}$ and $\Delta m_t^{Y_f}$.

When F is optimistic regarding at least one or both of the countries' growth rates, his leverage constraint binds, and his true expectations are not accurately reflected in his investment choices. Rewriting r_t^f from (25) in terms of investors' disagreement using (10) and (36) shows how the leverage constraint distorts the link between interest rates and expected consumption growth rates.[19]

$$\begin{aligned} r_{t,U}^f &= s_f^F m_{Yf,t}^{(F)} + (1-s_f^F) m_{Yf,t}^{(H)} - \sigma_{Yf,t}^2 \\ &= m_{Yf,t}^{(H)} - \sigma_{Yf}^2 + s_f^F \sigma_{Yf} \Delta m_t^{Yf} \end{aligned} \tag{41}$$

$$\begin{aligned} r_{t,LC}^f &= m_{Yf,t}^{(H)} - \sigma_{Yf}^2 + s_f^F \sigma_{Yf} \Delta \kappa_t^f \\ &= r_t^U + s_f^F \sigma_{Yf} (\vec{\sigma}_{S,t}^{-1} \upsilon_t \mathtt{I})_{\text{el.2}}, \end{aligned} \tag{42}$$

where $(\cdot)_{\text{el.2}}$ denotes the 2nd element of the vector $(\cdot)$. (42) shows that when the constraint binds, the induced reallocation of F's portfolio puts downward pressure on interest rates. $(\vec{\sigma}_{S,t}^{-1} \upsilon_t)_{\text{el.2}}$, reflects the wedge that the constraint drives between F's true disagreement about consumption growth rates and those reflected in his portfolio. This term is negative whenever the constraint binds, F holds fewer risky assets than he would optimally like to. There are two ways to see the intuition: as F is constrained from showing his true (optimistic) beliefs in his portfolio, the expected consumption growth rates that are implied by $r_{t,LC}^f$ are lower than true expected growth rates. Alternatively, one can consider the portfolio side: F's leverage under the binding constraint

[19]The change to r_t^h follows analogously.

is lower than it would be absent the constraint, under $r^f_{t,U}$. Lower effective demand for leverage reduces its price.

The magnitude of this distortion depends on three elements. First, the level of η. The amount of leverage F is permitted to take captures the 'strictness' of the constraint. Second, disagreement parameters $\Delta m_t^{Y_h}$ and $\Delta m_t^{Y_f}$. These determine how much of the economy's risk F would optimally carry in an unrestricted market, and therefore capture how 'severely' the constraint affects F. Third, the covariance structure of asset markets, $\vec{\sigma}_{S,t}$. Covariance determines how F will optimally use the other available assets to construct the best possible substitute portfolio that minimizes the impact of the constraint. This interaction creates a correlation between interest rates and stock market volatilities beyond the contribution of fundamental risk to interest rates.

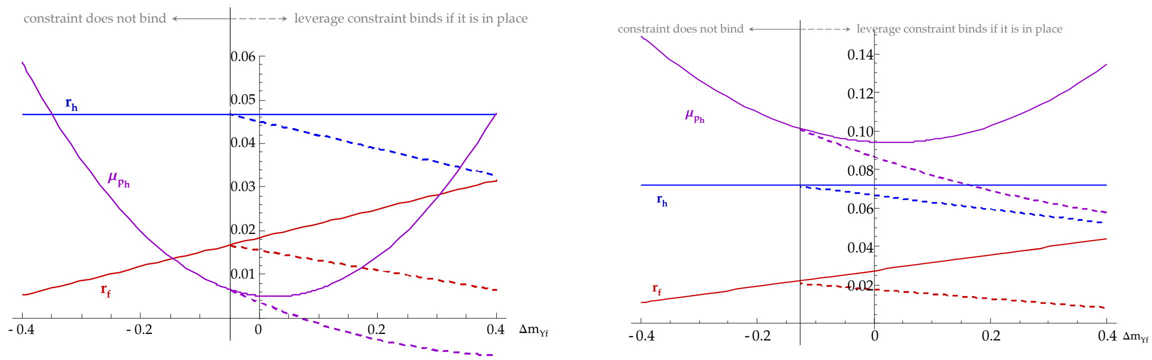
While $r^f_{t,U}$ depends positively on $\Delta m_t^{Y_f}$, it is independent of $\Delta m_t^{Y_h}$. This is not the case in the constrained equilibrium.

$$\frac{\partial r^f_{t,LC}}{\partial \Delta m_t^{Y_h}} = s^F_f \sigma_{Yf} \frac{\partial(\vec{\sigma}^{-1}_{S,t} \upsilon_t \mathtt{I})_{\text{el.2}}}{\partial \Delta m_t^{Y_h}} = \frac{-(1+\eta) s^F_f \sigma_{Yh} \sigma^2_{Yf} \sigma^2_\alpha}{\left((\eta-1)^2 \lambda_t^2 (\alpha_t^H + \alpha^F - 1)^2 \sigma^2_{Yh} \sigma^2_{Yf} + (\sigma^2_{Yh} + \sigma^2_{Yf}) \sigma^2_\alpha\right)} < 0,$$

$$\frac{\partial r^f_{t,LC}}{\partial \Delta m_t^{Y_f}} = \frac{\partial r^f_{t,U}}{\partial \Delta m_t^{Y_f}} + s^F_f \sigma_{Yf} \frac{\partial(\vec{\sigma}^{-1}_{S,t} \upsilon_t^{LC})_{\text{el.2}}}{\partial \Delta m_t^{Y_f}} = s^F_f \sigma_{Yf} \left(1 - \frac{(1+\eta)\sigma^2_{Yh}\sigma^2_\alpha}{(\eta-1)^2 \lambda_t^2 (\alpha_t^H + \alpha^F - 1)^2 \sigma^2_{Yh} \sigma^2_{Yf} + (\sigma^2_{Yh} + \sigma^2_{Yf}) \sigma^2_\alpha}\right) > 0.$$

As differences in beliefs $\Delta m_t^{Y_h}$ or $\Delta m_t^{Y_f}$ rise, F would optimally take on more stock market risk in this economy than he is able to. This unrealized demand for borrowing pushes down interest rates. Because F is constrained in his *joint* stock holdings, a sudden change in optimism about either country can trigger this effect. The constraint transmits effects of investor heterogeneity about one country's fundamentals to other countries' interest rates. In an economy with investment frictions, interest rates are more sensitive to other countries' investment opportunities than would be implied by frictionless models.

Changing the stringency of the imposed constraint η —essentially regulatory action—has the predictable impact. $\partial r_t^{LC}/\partial \eta > 0$ whenever the constraint binds. Lowering η implies a change in regulation that forces F to liquidate part of his portfolio to lower his leverage. Intuitively, as known from models with a single 'world' riskfree bond, this leads to a higher demand for the risk-free bond, markets clear at a lower interest rate r_t^f.



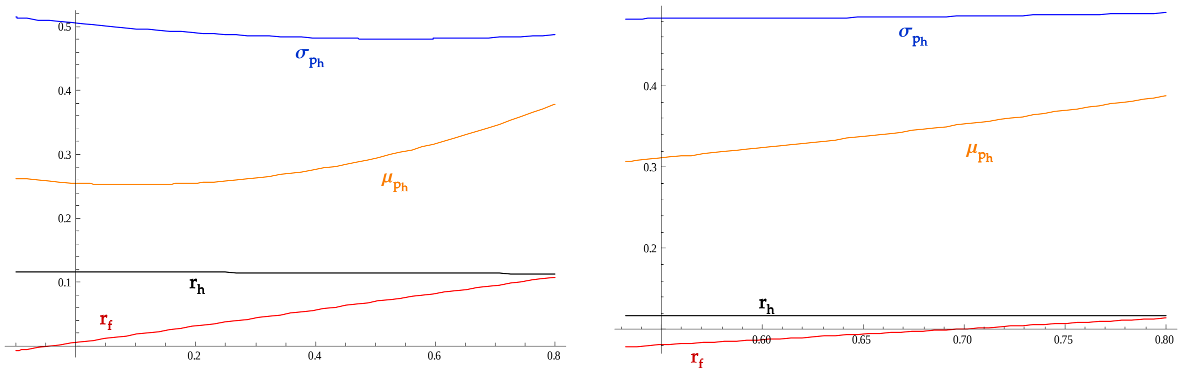
(a) $\Delta m_t^{Y_h} = 0.2$ (small)

(b) $\Delta m_t^{Y_h} = 0.5$ (large)

Figure 3: **UIP Violation—Unconstrained and Constrained** *home* and *foreign* interest rates r_t^h and r_t^f as well as expected appreciation of the *home* currency p_t^h, in the unconstrained (solid) and constrained case (dashed).

Fig.(3) illustrates the effect of a suddenly imposed leverage constraint on interest and exchange rates. Comparing the dashed lines in graphs (a) and (b) indicates the effect of suddenly imposing a leverage constraint on a previously unconstrained economy. It shows r_t^h, r_t^f and expected appreciation of the *home* curreny, both for the unconstrained case U (solid), as well as for the leverage-constrained case LC (dashed).The immediately binding leverage constraint would lead to the sudden drop in exchange rate expectations μ_{p_h}, moving against carry traders. Brunnermeier, Nagel, and Pedersen (2008) show that currency pairs that exhibit profitable carry trade opportunities tend to have a negatively skewed distribution. This is interpreted as 'currency crash risk': when a currency depreciates, volatility is higher. Although distributions here are not skewed, this downward adjustment of expected returns has a similar effect if such an event were part of a data sample.

Although the above shows that suddenly binding constraints can indeed affect currency markets in a way that poses a risk for investors with carry trade positions, comparing panels (a) and (b) in fig. (4) shows that negative skewness of the high-interest rate currency arises only when a very strict constraint is in place, and the effect is relatively weak. As investors' assessment of *foreign* country's investment opportunities changes, expected appreciation and volatility move in opposite



(a) $\eta = 1.1$ (strict leverage constraint)

(b) $\eta = 2.5$ (lenient leverage constraint)

Figure 4: **Negative 'Skew' Due to Funding Constraint** Only when constrained investor F is optimistic about investment opportunities in the high-interest-rate country will the exchange rate p_t^h be negatively skewed: As $\Delta m_t^{Y_f}$ falls, exchange rate volatility σ_{p_h} rises and expected appreciation μ_{p_h} falls.

directions. Both graphs are plotted for $\Delta m_t^{Y_h} = 0.8$, indicating that the *foreign* investor expects high growth rates in the *home* country.

Indeed, negative skewness only arises for high levels of $\Delta m_t^{Y_h}$; high interest rates can only be sustained by high expectations about economic growth. In panel (a), volatility rises and expected appreciation falls—the left tail of the negative skew—as $\Delta m_t^{Y_f}$ falls. The fairly equal sharing of *foreign* market risk while *home* risk becomes more unevenly shared produces a lopsided exposure, which is reflected in exchange rates.

This effect occurs when the leverage constraint η is low, indicating a strict constraint. The stricter the constraint is, the more F's portfolio is distorted for given beliefs about growth rates. The magnitude of this distortion creates the negative skew. Volatility, as the square root of variance, is affected by the absolute magnitude of investor heterogeneity $\Delta m_t^{Y_h}$ and $\Delta m_t^{Y_f}$, whereas beliefs about the two countries growth rates will have opposite directional effects on expected exchange rate movements μ_{p_h}.

This, albeit tentative, result suggests that negative skewness found in currency options is not necessarily the result of concern about a crash in economic fundamentals of high-interest rate countries, but could also be an indicator of anticipated fluctuations in sentiment and disparity of international risk sharing.

IV Conclusion

The paper studies a two-country open economy model that endogenously generates currency risk premia.

The model shows that Uncovered Interest Parity will be violated under certain conditions, giving rise to the 'carry trade'—the profits from investing in a high-interest-rate bond with money borrowed in a low-interest-rate country will not be offset by a commensurate depreciation of the high-interest currency.

Interest rate differentials reflect investors beliefs about the ability of the countries' respective output to keep up with the growth in demand. However, due to integrated goods as well as financial markets, demand for goods is sensitive to endogenous changes in the wealth distribution across investors. If demand for a country's good relative to its supply is risky, its currency is a bad hedge against systematic risk, its return must be high, the currency appreciates on average. These conditions, under which UIP is violated, are more likely to occur when the investor foreign to this country is rich and carries more aggregate risk.

This suggests that one should find profitable carry trade in situations where the high interest rate country is a growing economy that can, in expectation, keep up with future demand growth, but where demand for its good is erratic due to being dependent on exports to a country whose wealth is sensitive to stock market risk.

The exchange rate risk premium is simply compensation for systematic risk in a Gaussian economy, but the model is consistent with finding skewness in the time series of exchange rates. The parameters determining exchange rates are endogenous, and thus time-varying. For the carry trade skewness is detrimental if the volatility of the exchange rate rises just as the exchange rate moves against the carry trade. This covariance is negative, creating the impression of a negative skew in the data, if aggregate risk is unevenly distributed across the two agents. Thus, the same condition that makes a UIP violation (i.e. a profitable carry trade) between two countries more likely, also generates exchange rate dynamics that are consistent with finding a skewed distribution in the data.

This paper studies exchange rate dynamics in open economy with unsegmented goods as well financial markets in order to better understand currency premia and the conditions under which carry trades are profitable. Many currency pairs where this is the case have reasonably integrated markets, therefore separating out the effects of segmentation to provide hypotheses on these markets is valuable. Export dependency of high-growth countries, as well as the allocation of aggregate risk across investors is shown to have a significant impact on currency risk premia.

Appendix

A Alternative Constraint: Limit on non-domestic stockholding

conceptually similar results arise in the presence of a different type of constraint: a limit on F's investment into the stock market abroad, S_t^h. Where the leverage constraint allows the investor the freedom to optimize the allocation among stocks even if not the total amount of stock holdings, this type of constraint is more one-sided. This makes it more difficult to construct an optimal portfolio of stocks, but conversely leaves more freedom in other, unconstrained assets, to compensate for the constraint. Referring to this constraint as ND—a restriction on non-domestic stocks—the restriction is formalized as follows.

$$\text{non-domestic stockholding constraint:} \qquad \mathtt{I}_{ND}^{\top}\pi_{F,t} \leqslant \varphi \tag{43}$$

where $\mathtt{I}_{ND} = [1,0,0]^{\top}$, and $\pi_{i,t} = [\pi_{i,t}^{S_h}, \pi_{i,t}^{S_f}, \pi_{i,t}^{B_h}]^{\top}$ is the vector of portfolio holdings of investor i in both stocks and the *home* bond.

case ND: equilibrium when investor F faces a constraint on holdings of S_t^h

$$\pi_{Ht} = (\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(H)} - r_t\mathbf{1}\right), \tag{44}$$

$$\pi_{Ft} = \begin{cases} (\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right) + (\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\upsilon_t^{ND}\mathtt{I}_{ND} & \text{if } \upsilon_t^{ND} < 0 \\ (\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right) & \text{otherwise} \end{cases} \tag{45}$$

where $\upsilon_t^{ND} = \min\left(\frac{\varphi - \mathtt{I}_{ND}^{\top}(\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right)}{\mathtt{I}_{ND}^{\top}(\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\mathtt{I}_{ND}}, 0\right)$, ie.

$$\upsilon_t^{ND} = \begin{cases} \frac{-\sigma_{Y_h}\sigma_\alpha^2\left[\Delta m_t^{Y_h} - \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Y_h}\right]}{\left(\varphi\lambda_t(\alpha_t^H + \alpha^F - 1) + (1-\alpha_t^H)(\alpha_t^H + (1-\alpha^F)\lambda_t)\right)^2\sigma_{Y_h}^2 + \sigma_\alpha^2} & \text{if } \Delta m_t^{Y_h} > \left(\varphi\,(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Y_h}, \\ 0 & \text{otherwise.} \end{cases} \tag{46}$$

The collateral adjustment in this case is $\delta(\upsilon_t^{ND}) = -\varphi\upsilon_t^{ND}$.

A.1 r_t When Non-Domestic Holdings Are Constrained

$$r_t^{ND} = r_t^U + s_f^F \sigma_{Yf} (\vec{\sigma}_{S,t}^{-1} \upsilon_t^{ND})_{\text{el.2}} - s_f^F \delta(\upsilon_t^{ND}) \tag{47}$$

where $(\cdot)_{\text{el.i}}$ denotes the i'th element of the vector $(\cdot)$.

In scenario ND, $(\vec{\sigma}_{S,t}^{-1}\upsilon_t^{ND})_{\text{el.2}} = 0$. Being restricted only in one particular asset, S_t^h, sufficient alternative securities remain such that the two investors can efficiently trade the other source of fundamental risk, the *foreign* production risk. Accordingly, investors' beliefs about this unrestricted market will be correctly reflected by portfolios: $\Delta\kappa_t^f = \Delta m_t^{Y_f}$. F reallocates part of his wealth into a combination of the *home* bond B_t^h, providing exposure to exchange rate risk that he would otherwise carry through the inaccessible stock S_t^h, and the remaining risky asset, S_t^f.

However, the collateral adjustment $\delta(\upsilon_t^{ND}) = \frac{\varphi\sigma_{Yh}\sigma_\alpha^2\left[\Delta m_t^{Y_h} - \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Yh}\right]}{\left(\varphi\lambda_t(\alpha_t^H + \alpha^F - 1) + (1-\alpha_t^H)(\alpha_t^H + (1-\alpha^F)\lambda_t)\right)^2 \sigma_{Yh}^2 + \sigma_\alpha^2} > 0$ affects the precautionary savings motive. A constraint on long positions implies that this (rather optimistic) investor would like to invest more and thus feels precluded from participating in future growth, which he partly compensates for by investing more of his wealth into the riskfree asset than would be the case for a standard agent of his utility; this lowers the interest rate.

Firstly, a constraint is tighter when the imposed investment limit, φ in case ND, is lowered: when the constraint binds, F is forced to liquidate part of his holdings of S_t^h. Secondly, a given constraint is tighter when it endogenously binds more severely: the constraint distorts the portfolio more, desired and realized portfolio are very different. This is the case when, for a fixed level of φ, investor F wants to holds a large long position in stock S_t^h. Investor beliefs and changes therein over time capture this latter effect. Higher $\Delta m_t^{Y_h}$ implies that investor F is more bullish about investment opportunities in *home* country, but cannot purchase more of the stock. So although holdings of the restricted stock are not explicitly affected, the constraint will feed back into other security markets.

First, consider a tightening of constraints in the latter sense: how is the interest rate affected when a constraint that is in place starts binding more severely. Recall from (**??**) that r_t^U is independent of beliefs regarding Y_t^h's growth rate; the riskfree asset provides one unit of the numeraire consumption good in the future, and is therefore independent of the consumption risks associated with other goods.

Substituting equilibrium terms from proposition 1 into (47) shows that this independence no longer holds when the *foreign* investor is bound by his constraint:

$$\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Y_h}} = \frac{-\varphi s_f^F \sigma_{Yh} \sigma_\alpha}{\left(\varphi \lambda_t(\alpha_t^H + \alpha^F - 1) + (1-\alpha_t^H)(\alpha_t^H + (1-\alpha^F)\lambda_t)\right)^2 \sigma_{Yh}^2 + \sigma_\alpha^2} < 0.$$

The interest rate r_t^{ND} falls as the *foreign* investor is constrained more severely. His portfolio reallocation increases demand for the riskfree bond, lowering interest rates.

Having three other assets available to compensate for the restriction of S_t^h holdings, the fundamental risk of the *foreign* country can be optimally shared among investors, in accordance with their true beliefs about growth rates. This implies that the risk-free rate's sensitivity to this risk is identical to that in an identical but unconstrained economy: $\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Yf}} = \frac{\partial r_t^{U}}{\partial \Delta m_t^{Yf}} = s_f^F \sigma_{Yf} > 0$: as disagreement about the *foreign* growth rate increases, the aggregate expected growth rate of consumption increases, putting upwards pressure on interest rate.

The model does not make explicit assumptions about which country's border the constraints are imposed at. The restriction on S_t^h for investor F could be due to *home* keeping foreign investment out or indeed due to the *foreign* government attempting to keep money in the country. The former type of restrictions are more commonly considered when thinking of the liberalization efforts during the 1980's and 1990's. The latter however also exist, for example in Argentina during a period in the early 2000s, and China still retains some restrictions on capital leaving local markets.

Now consider the other notion of 'tightening' constraints: a regulatory decision to lower the level φ. In contrast to the situation where the investor was more severely affected by the restriction due to his beliefs, this type of regulatory change leads to trade in the restricted asset. Investor F, already constrained, now has to sell some of his holdings in the *home* stock and reallocate them elsewhere. The effect of this reallocation on interest rates is ambiguous:

$$\frac{\partial r_t^{ND}}{\partial \varphi} \begin{cases} > 0 & \text{if } \varphi > \sqrt{\frac{(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Yf}^2+\sigma_\alpha^2}{\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Yf}^2}} \\ < 0 & \text{if } \varphi < \sqrt{\frac{(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Yf}^2+\sigma_\alpha^2}{\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Yf}^2}} \; \& \\ & \Delta m_t^{Y_h} > \left(\varphi\,(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Yh} + A(\varphi) \end{cases}$$

where $A(\varphi)$ is a positive term, the details of which can be found in the appendix.

Crucially, whether tightening investment restrictions has a positive or negative effect on interest rates itself depends on how severely it was binding at the time the change is implemented. This interaction effect has not been discussed in any detail within both the theoretical and the empirical literature.

Consider the first of these two cases above: when φ is relatively lenient, changing the regulation to make this limit of *home* stockholding more strict will have the intuitive effect: As the constraint is tightened and F is forced to liquidate some of his holdings of S_t^h, he reallocates some of these freed funds into substitute assets, including his local bond market B_t^f. The increased demand for bonds means markets will clear at lower interest rates.

Conversely, tightening the investment limit has the opposite effect on interest rates when the constraint is already quite strict (φ low), and it binds severely ($\Delta m_t^{Y_h}$ high). A very strict limit implies that to take on the desired amount of risk, F has to hold large positions in the substitute assets. These, however, are imperfectly correlated with S_t^h. The investor faces the trade off between sacrificing his total amount of risk exposure or the diversification in his portfolio.

The more severely the constraint binds, the more he is willing to sacrifice diversification in response to a sudden change in regulation. He will tilt his portfolio more towards the alternative risky assets, to retain sufficient exposure to economic risk. Participating in the (high) expected growth of the economy through sufficient risk exposure overrides a risk-averse agent's desire to hold a diversified portfolio. The resulting drop in F's demand for bonds raises interest rates.

This trade off made by the restricted investor between diversification and overall risk exposure is not unique to this particular type of constraint. The next section will show how this intuition plays out in the setting of a leverage constraint. It illustrates clearly that regulatory intervention, often sought at times of high disagreement and uncertainty, can have counterintuitive results which must be considered.

B Optimal Consumption

Investors H and F maximize their respective expected utility, subject to budget constraints. Equilibrium is established by maximizing the aggregated utility function

$$U(C_H, C_F) = u_H\left(C^h_{H,t}, C^f_{H,t}\right) + \lambda_t u_F\left(C^h_{F,t}, C^f_{F,t}\right)$$

where

$$\begin{aligned} u_H\left(C^h_{H,t}, C^f_{Ht}\right) &= \alpha^H_t \log C^h_{Ht} + (1-\alpha^H_t)\log C^f_{H,t}, \\ u_F\left(C^h_{F,t}, C^f_{Ft}\right) &= (1-\alpha^F)\log C^h_{Ft} + \alpha^F \log C^f_{F,t}, \end{aligned}$$

and $\lambda_t = \frac{y_H \xi^H_t}{y_F \xi^F_t}$, the ratio of investors' state price densities.

FOC of optimal consumption of goods $j = h, f$, of investors $i = H, F$: $u^i_{C^j}(\cdot) = \frac{\partial u_i\left(C^i_{it}, C^j_{it}\right)}{\partial C^j_{it}} = y_i p^j_t \xi^i_t$, where p^j_t is the relative price of good j, ξ^i_t is investor i's state price density and y_i the associated Lagrange multiplier, reflecting initial endowment.

	investor H:	*investor F:*
good h:	$\frac{\alpha^H_t}{C^h_{Ht}} = y_H p^h_t \xi^H_t$	$\frac{1-\alpha^F}{C^h_{Ft}} = y_F p^h_t \xi^F_t$
good f:	$\frac{1-\alpha^H_t}{C^f_{Ht}} = y_H p^f_t \xi^H_t$	$\frac{\alpha^F}{C^f_{Ft}} = y_F p^f_t \xi^F_t$

Market clearing requires $\sum_i C^j_i = Y^j$ for both goods $j = h, f$, giving equilibrium total consumption in section 4.

C Optimal Wealth

Current wealth is an appropriately discounted value of all future consumption levels. Log utility in a finite horizon economy implies that both investors will consume a fixed portion of their wealth each period, as a function of the time remaining. The below is described for investor H, analogous values for investor F follow directly.

$$X_t^H = \frac{1}{\xi_t^H} E\left[\int_t^T \left(\xi_s^H p_s^h C_{Hs}^h + \xi_s^H p_s^f C_{Hs}^f\right) ds\right]$$

From FOC above, $\frac{\alpha_t^H}{y_H} = C_{Ht}^h p_t^h \xi_t^H$ and $\frac{1-\alpha_t^H}{y_H} = C_{Ht}^f p_t^f \xi_t^H$ holds, therefore:

$$X_t^H = \frac{1}{\xi_t^H} E\left[\int_t^T \left(\frac{\alpha_s^H}{y_H} + \frac{1-\alpha_s^H}{y_H}\right) ds\right] = \frac{1}{y_H \xi_t^H}(T-t).$$

Linking wealth X_t^i back to consumption above gives

$$\begin{aligned} X_t^H &= C_{Ht}^h \cdot \frac{p_t^h}{\alpha_t^H}(T-t) = C_{Ht}^f \cdot \frac{p_t^f}{1-\alpha_t^H}(T-t), \\ X_t^F &= C_{Ft}^h \cdot \frac{p_t^h}{1-\alpha^F}(T-t) = C_{Ft}^f \cdot \frac{p_t^f}{\alpha^F}(T-t). \end{aligned}$$

D Relative Goods Prices

The relative price of the two goods is determined by their relative marginal utilities, which must be equal across the two agents, since both are faced with identical prices for goods, there are no frictions in goods markets: $\bar{p}_t = \frac{p_t^f}{p_t^h} = \frac{u_{C^f}^i(\cdot)}{u_{C^h}^i(\cdot)}$. The basket of goods $\beta p_t^h + (1-\beta) p_t^f = 1$ defines the numeraire. $\beta \in [0,1]$ and represents the weight of the *home* good in the basket. This weight does not represent either agent's de facto consumed basket. The levels of stock prices will be affected by the chosen β, but the relation between the two stocks will not be. Interesting special cases include $\beta = 0$, $\beta = 1$ or $\beta = \alpha^F$, denoting Y_t^f, Y_t^h or F's true consumption basket as the numeraire, respectively. The main insights from the paper are not sensitive to the choice of β.

Using the equilibrium marginal utilities from market clearing restrictions $\sum_i C_i^j = Y^j$ for goods

$j = h, f$ gives:

$$\bar{p}_t = \frac{p_t^f}{p_t^h} = \frac{u^H_{C^f}(\cdot)}{u^H_{C^h}(\cdot)} = \frac{y_H p_t^f \xi_t^H}{y_H p_t^h \xi_t^H} = \frac{(1-\alpha_t^H) + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{Y_t^f}.$$

The dynamics of relative goods prices $\bar{p}_t$ follow

$$\begin{aligned} d\bar{p}_t &= (\cdot)dt + \frac{1-\alpha_t^H + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{1}{Y_t^f} dY_t^h - \frac{1-\alpha_t^H + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{(Y_t^f)^2} dY_t^f - \\ & - \frac{\lambda_t + 1}{(\alpha_t^H + (1-\alpha^F)\lambda_t)^2} \frac{Y_t^h}{Y_t^f} d\alpha_t^H + \frac{2\alpha_t^H - 1}{(\alpha_t^H + (1-\alpha^F)\lambda_t)^2} \frac{Y_t^h}{Y_t^f} d\lambda_t. \end{aligned}$$

E Auxiliary Market: Portfolio Choice in Constrained Markets

The constraints studied are limitations on the fraction of wealth $\pi_{i,t}^j$ that investor i places into one or more assets j. I assume that portfolio positions $\pi_{i,t}^j$ in assets $j = S_t^h, S_t^f, B_t^h, B_t^f$ are constrained to lie in a closed, convex, non-empty set K that contains the origin. The analysis here is based on the methodology developed in Cvitanic and Karatzas (1992).

The martingale analysis of incomplete markets requires the construction of a fictitious market that fictitiously augments the market parameters of the original constrained market. Under these augmented market parameters, the constrained investor will optimally choose a portfolio permissible within the constraints. This is then the optimal portfolio also under the original, constrained market.[20]

The set of admissible trading strategies is defined by the set K, the support function is $\delta(\upsilon_t^i) \equiv \delta(\upsilon_t^i | K) \equiv \sup\left(-\pi_{i,t}^\top \upsilon_t^i : \pi_{i,t} \in K\right)$ and the barrier cone of the set $-K$ is defined as $\bar{K} \equiv \left\{\upsilon_t^i \in \mathbb{R}^2 | \delta(\upsilon_t^i) < \infty\right\}$. υ_t^i is a square-integrable, progressively measurable process taking values in $\bar{K}$ to ensure boundedness.

Investor F's state price density adjust to reflect these augmented market perceptions due to the constraints:

$$d\xi_t^F = -\left(r_t + \delta(\upsilon_t^F)\right) \xi_t^F dt - \kappa_t^{F\top} \xi_t^F d\vec{W}_t^{(F)}, \tag{48}$$

[20]This setting is a straightforward application of that in Cvitanic and Karatzas (1992), and it can be easily shown that their convex duality approach for convex constraint sets holds here.

where investor F's adjusted market price of risk is $\vec{\kappa}_t^F = (\sigma_{S,t}^{-1})\left(m_{S,t}^{(F)} + \upsilon_t^F \iota_F - r_t \mathbf{1}\right) = \kappa_{o,t}^F + \sigma_{S,t}^{-1}\upsilon_t^F$. $\kappa_{o,t}^F$ represents the market price of risk that the investor would base his portfolio decisions on, i.e. those reflecting his true beliefs. The second term, $+\sigma_{S,t}^{-1}\upsilon_t^F$, adjusts the market price of risk s.t. the investor does not violate his constraint, and at the same time captures the market price of risk that will be reflected in portfolio choice and thus equilibrium market prices.

F State Price Density

Investor H consumes a fraction $\frac{\alpha_t^H}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ of good Y_t^h and a fraction $\frac{1-\alpha_t^H}{1-\alpha_t^H + \alpha^F\lambda_t}$ of good Y_t^f. This and equilibrium relative prices $\bar{p}_t$ gives

$$\xi_t^H = \beta \frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{y_H Y_t^h} + (1-\beta)\frac{1-\alpha_t^H + \alpha^F\lambda_t}{y_H Y_t^f}. \tag{49}$$

Analogously, investor F consumes a fraction $\frac{\lambda_t(1-\alpha^F)}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ of good Y_t^h and a fraction $\frac{\lambda_t\alpha^F}{1-\alpha_t^H + \alpha^F\lambda_t}$ of good Y_t^f:

$$\xi_t^F = \beta \frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{\lambda_t y_F Y_t^h} + (1-\beta)\frac{1-\alpha_t^H + \alpha^F\lambda_t}{\lambda_t y_F Y_t^f}. \tag{50}$$

G Asset Valuation

Proof of Proposition 1: The proof follows closely that in Schornick (2009), under the simpler situation that H does not face a constraint.

Market clearing in asset markets requires

$$S_t^h + S_t^f = X_t^H + X_t^F = p_t^h Y_t^h (T-t) + p_t^f Y_t^f (T-t). \tag{51}$$

Each asset $j = h, f$ is valued as the sum of discounted dividends, taking into account the effects of future binding constraints — the second integral in the equation below.

$$S_t^j = \frac{1}{\xi_t^H} E_t\left[\int_t^T \xi_s^H p_s^j Y_s^j \, ds\right] \qquad j = h, f.$$

Using $\frac{1}{p_t^h \xi_t^H} = \frac{Y_t^h y_H}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ and goods market clearing, as well as $\lambda_t = \frac{y_H \xi_t^H}{y_F \xi_t^F}$ in the pricing function of S_t^h:

$$S_t^h = p_t^h Y_t^h (T-t) + \frac{p_t^h Y_t^h}{\alpha_t^H + (1-\alpha^F)\lambda_t}(1-\alpha^F)\left[E_t \int_t^T \lambda_s ds - \lambda_t (T-t)\right] \tag{52}$$

$$S_t^f = p_t^f Y_t^f (T-t) + \frac{p_t^f Y_t^f}{1-\alpha_t^1 + \alpha^2 \lambda_t}\alpha^F \left[E_t \int_t^T \lambda_s ds - \lambda_t (T-t)\right] \tag{53}$$

Under the constraints on investor F $d\lambda_t$ is a supermartingale under all possible equilibria. Therefore,

$$\begin{aligned} S_t^h &= p_t^h Y_t^h (T-t), \\ S_t^f &= p_t^f Y_t^f (T-t), \end{aligned} \tag{54}$$

where p_t^h and p_t^f can be rewritten in terms of $\bar{p}_t$.

H Interest Rate Effects

In section III.A the sensitivity of the interest rate in scenario ND with respect to restriction parameter φ is detailed. $A(\varphi) = \frac{(1+\lambda_t)\varphi\sigma_{Y_h}\left(\left(\varphi\lambda_t(\alpha_t^H+\alpha^F-1)+(1-\alpha_t^H)(\alpha_t^H+(1-\alpha^F)\lambda_t)\right)^2\sigma_{Y_h}^2+\sigma_\alpha^2\right)}{-\varphi^2\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_h}^2+(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Y_h}^2+\sigma_\alpha^2} > 0$

In scenario LC, the function that determines the sign of $\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Y_f}}$ is

$B(\cdot) = \frac{\sigma_\alpha^2\sigma_{Y_h}^2 \pm \sqrt{\left(\sigma_\alpha^2\sigma_{Y_h}^2+4\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_f}^2(\sigma_{Y_h}^2-\sigma_{Y_f}^2)\right)\sigma_\alpha^2\sigma_{Y_h}^2}}{2\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_f}^2\sigma_{Y_h}^2}$.

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